Online Appendix

The Millennium Peak in Club Convergence -

A New Look at Distributional Changes

in the Wealth of Nations

Melanie Krause*

July 5, 2016

1 Proof of Theorem 3 (Polarization Comparison)

Proof. (a) In a bimodal setting, the reaction of CB, P_W and P_{ER} to a *ceteris paribus* increase (decrease) in between-cluster separation or within-cluster concentration follows directly from the definition of the measures, see Corollaries 2 and 3 in Section 3 for CB as well as Esteban and Ray (2012) for P_W and P_{ER} .

(b) Scale Invariance:

- The standardized values on which CB is calculated are unaffected by a multiplicative factor c in $z = cx$: $\frac{z-\mu_z}{\sigma_z} = \frac{cx-c\mu_x}{c\sigma_x} = \frac{x-\mu_x}{\sigma_x}$.
-
$$P_W(z) = 2\frac{\mu_z}{m_z}\left(1 - 2L_z(0.5) - Gini_z\right) \quad (1)$$
with $z = cx$ can be reduced to $P_W(x)$ by making use of $L_z = L_x$, $Gini_z = Gini_x$, $\mu_z = c\mu_x$ as well as $m_z = cm_x$.[1]
- In
$$P^{\alpha}_{ER}(z, w) = \int\int f(z)^{1+\alpha} f(w)|z - w|dzdw, \quad (2)$$
one can make use of
$$z = u(x) \quad \Longrightarrow \quad f_z(z) = \left|\frac{\partial(u^{-1}(z))}{\partial z}\right| \cdot f_x(u^{-1}(z)) \quad (3)$$

*Correspondence address: University of Hamburg, Department of Economics, Von-Melle-Park 5, 20146 Hamburg, Germany; E-mail: *melanie.krause@wiso.uni-hamburg.de*

[1] This is also the reason why P_W is the same both on raw and mean-standardized data: Mean-standardization can be seen as scaling by a constant $c = \frac{1}{\mu}$.

to obtain the density $f(cx)$ (and equivalently $f(cy)$). With $z = cx = u(x)$, $f(z) = \frac{1}{c} f(x)$ as well as $dz = c \cdot dx$, one arrives at

$$P^{\alpha}_{ER}(z, w) = \int \int \Big(\frac{1}{c} f(x)\Big)^{1+\alpha} \frac{1}{c} f(y) |cx - cy| c^2 dx dy = c^{1-\alpha} P_{ER}(x, y).$$

This is the homogeneity of degree zero property Duclos et al. (2004) point out: Mean-standardizing ($c = \frac{1}{\mu}$) the data scales P_{ER} by $\mu^{\alpha-1}$. It can directly be used to prove the second part of the statement:

$$P_{ER}\Big(\frac{z}{\mu_z}\Big) = \mu_z^{\alpha-1} P_{ER}(z) = (c\mu_x)^{\alpha-1} c^{1-\alpha} P_{ER}(x) = \mu_x^{\alpha-1} \mu_x^{1-\alpha} P_{ER}\Big(\frac{x}{\mu_x}\Big) = P_{ER}\Big(\frac{x}{\mu_x}\Big)$$

(c) Invariance to Absolute Income Changes:

- The standardized values on which CB is calculated are unaffected by an additive constant a in $z = x + a$: $\frac{z-\mu_z}{\sigma_z} = \frac{x+a-(\mu_x+a)}{\sigma_x} = \frac{x-\mu_x}{\sigma_x}$.
- Similar to part (a), $P_W(z)$ with $z = x + a$ can be expressed in terms of $P_W(x)$, using $\mu_z = \mu_x + a$ and $m_z = m_x + a$. We still need an expression of $Gini_z$ in terms $Gini_x$. To this end, recall the general formula for the Gini coefficient as the relative mean absolute difference

$$Gini_x = \frac{E|x_i - x_j|}{2\mu_x} \tag{4}$$

so that

$$Gini_z = \frac{E|x_i + a - (x_j - a)|}{2(\mu_x + a)} = \frac{E|x_i - x_j|}{2(\mu_x + a)} = \frac{\mu_x}{\mu_x + a} Gini_x. \tag{5}$$

Now, with $L_x(0.5) = \frac{\sum_{j=1}^{0.5n} x_j}{n\mu_x}$ one can write $L_z(0.5)$ in terms of $L_x(0.5)$:

$$L_z(0.5) = \frac{\sum_{j=1}^{0.5n} z_j}{\sum_{k=1}^{n} z_k} = \frac{\sum_{j=1}^{0.5n} x_j + 0.5 \cdot n \cdot a}{n(\mu_x + a)} = \frac{L_x(0.5) \cdot n \cdot \mu_x + 0.5 \cdot n \cdot a}{n(\mu_x + a)} \tag{6}$$

$$\Longrightarrow P_W(z) = 2\frac{\mu_x + a}{m_x + a}\Big(1 - 2\frac{L_x(0.5) \cdot \mu_x + 0.5 \cdot a}{\mu_x + a} - \frac{\mu_x}{\mu_x + a} Gini_x\Big), \tag{7}$$

which after some algebra simplifies to $P_W(z) = \frac{m_x}{m_x+a} \cdot P_W(x)$. Increasing all incomes by a positive amount will thus decrease bipolarization.

- Following the same steps as in part (b) with the transformation $z = x + a$, one can directly see that the densities and differences involved do not change, hence $P_{ER}(z) = P_{ER}(x)$. This implies for the mean-standardized data:

$$P_{ER}\Big(\frac{z}{\mu_z}\Big) = \mu_z^{\alpha-1} P_{ER}(z) = \Big(\frac{1}{\mu_x + a}\Big)^{1-\alpha} P_{ER}(x) = \Big(\frac{\mu_x}{\mu_x + a}\Big)^{1-\alpha} P_{ER}\Big(\frac{x}{\mu_x}\Big)$$

(d) Dispersion Invariance:

- The standardized values on which CB is calculated are unaffected by a λ-squeeze or λ-dispersion $z = \lambda x + (1-\lambda)\mu_x$: $\frac{z-\mu_z}{\sigma_z} = \frac{\lambda x + (1-\lambda)\mu_x - \mu_x}{\lambda \sigma_x} = \frac{x-\mu_x}{\sigma_x}$.
- Similar to (b) and (c), with $z = \lambda x + (1-\lambda)\mu_x$ it holds that $\mu_z = \mu_x$, while $m_z \neq m_x$ unless the distribution is symmetric. Plugging

$$Gini_z = \frac{E|\lambda x_i + (1-\lambda)\mu_x - (\lambda x_j + (1-\lambda)\mu_x)|}{2\mu_z} = \lambda \frac{E|x_i - x_j|}{2\mu_x} = \lambda Gini_x \qquad (8)$$

and

$$L_z(0.5) = \frac{\sum_{j=1}^{0.5n} z_j}{\sum_{k=1}^{n} z_k} = \frac{\sum_{j=1}^{0.5n}(\lambda x_j + (1-\lambda)\mu_x)}{n\mu_x} = \lambda L_x(0.5) + 0.5(1-\lambda) \qquad (9)$$

into the formula for $P_W(z)$ yields $P_W(z) = \lambda \frac{m_x}{m_z} \cdot P_W(x)$.

- One can follow the same steps as in part (b) to show that $P_{ER}(z) = \lambda^{1-\alpha} P_{ER}(x)$ because $z = \lambda x + (1-\lambda)\mu_x$ induces the same changes to the density and differences as $z = cx$ with $c = \lambda$. For the mean-standardized data it holds:

$$P_{ER}\Big(\frac{z}{\mu_z}\Big) = \mu_z^{\alpha-1} P_{ER}(z) = \mu_x^{\alpha-1}\lambda^{1-\alpha} P_{ER}(x) = \lambda^{1-\alpha} P_{ER}\Big(\frac{x}{\mu_x}\Big) \qquad (10)$$

(e) Symmetry of the Polarization Measure - Swapping Rich and Poor:

- The transformation equals a reflection of the distribution along the vertical line at $\frac{x_L + x_U}{2}$. The density values of two points of the distribution are swapped, $f(z) =$

$f(x_U + x_L - x)$, which also holds for the standardized densities. The modality of the distribution - and hence CB - are unaffected by this symmetric reflection.

- For P_W one can proceed analogously to parts (b) to (d) and derive $\mu_z = x_L + x_U - \mu_x$,

$$Gini_z = \frac{E|x_L + x_U - x_i - (x_L + x_U - x_j)|}{2\mu_z} = \frac{E|x_i - x_j|}{2\mu_z} = \frac{\mu_x}{\mu_z} Gini_x \tag{11}$$

as well as

$$L_z(0.5) = \frac{-L_x(0.5) \cdot \mu_x + 0.5(x_L + x_U)}{x_L + x_U - \mu_x}. \tag{12}$$

Substitution into the formula yields $P_W(z) = \frac{m_x}{m_z} P_W(x)$.

- Following the same steps as in part (b) with the transformation $z = x_L + x_U - x$, one can see the densities and differences involved do not change and $P_{ER}(z) = P_{ER}(x)$. For the mean-standardized data:

$$P_{ER}\Big(\frac{z}{\mu_z}\Big) = (x_L + x_U - \mu_x)^{\alpha - 1} P_{ER}(x) = \Big(\frac{\mu_x}{x_L + x_U - \mu_x}\Big)^{1-\alpha} P_{ER}\Big(\frac{x}{\mu_x}\Big).$$

□

2 Overview of Countries in the Data Set

Country	Absolute Income per Cap. 1970	2011	Stand. Income Per Cap. 1970	2011	Growth per Year (1970-2011)
Albania	3747.64	7364.71	-0.1998	-0.3771	0.0166
Argentina	2950.94	14507.62	-0.3534	0.0988	0.0396
Australia	16750.87	38499.27	2.3062	1.6974	0.0205
Austria	12406.66	37282.53	1.4690	1.6164	0.0272
Bahamas	12045.06	19366.61	1.3993	0.4226	0.0117
Bangladesh	1364.96	1554.21	-0.6590	-0.7643	0.0032
Belgium	14252.76	35446.27	1.8248	1.4940	0.0225
Belize	3510.06	7366.64	-0.2456	-0.3770	0.0182
Benin	1153.63	1231.88	-0.6997	-0.7857	0.0016
Bhutan	1084.23	4607.02	-0.7131	-0.5609	0.0359
Bolivia	1629.12	4166.78	-0.6081	-0.5902	0.0232
Botswana	706.120	11810.75	-0.7860	-0.0809	0.0711
Brazil	3116.37	9294.53	-0.3215	-0.2485	0.0270
Burkina Faso	503.75	1051.52	-0.8250	-0.7978	0.0181
Bulgaria	3023.84	12906.67	-0.3393	-0.7800	0.0360
Burundi	558.01	490.14	-0.8145	-0.8352	-0.0032
Cambodia	1298.75	2347.91	-0.6718	-0.7114	0.0145
Cameroon	1233.60	1857.53	-0.6843	-0.7441	0.0100
Canada	16064.66	35344.87	2.1740	1.4872	0.0194
Cape Verde	965.64	4125.81	-0.7360	-0.5929	0.0361
Central African Republic	1032.93	617.29	-0.7230	-0.8267	-0.0125
Chad	1123.32	1851.12	-0.7056	-0.7445	0.0123
Chile	6336.88	15243.33	0.2992	0.1479	0.0216
China	966.92	8068.60	-0.7357	-0.3302	0.0531
Colombia	4025.07	8407.92	-0.1463	-0.3076	0.0181
Comoros	1166.41	921.28	-0.6973	-0.8064	-0.0057
Congo (Dem. Rep.)	836.55	290.63	-0.7609	-0.8485	-0.0255
Congo (Rep.)	1270.84	2426.87	-0.6772	-0.7061	0.0159
Costa Rica	5446.86	10123.36	0.1277	-0.1933	0.0152
Cote d'Ivoire	2363.19	1371.83	-0.4666	-0.7764	-0.0132
Cyprus	5797.19	28183.25	0.1952	1.0101	0.0393
Denmark	16978.34	35641.17	2.3501	1.5070	0.0183
Djibouti	5402.75	2391.99	0.1192	-0.7084	-0.0197
Dominican Republic	2705.56	8726.60	-0.4600	-0.2864	0.0290
Ecuador	2533.23	6828.09	-0.4339	-0.4129	0.0245
Egypt	905.40	4836.37	-0.7476	-0.5456	0.0417
El Salvador	816.88	1116.53	-0.7646	-0.7934	0.0077
Equatorial Guinea	353.50	9175.83	-0.8539	-0.2564	0.0827
Ethiopia	556.73	782.71	-0.8148	-0.8157	0.0083
Fiji	2951.07	4644.74	-0.3533	-0.5583	0.0111
Finland	13099.40	33747.33	1.6025	1.3808	0.0233
France	14512.68	31437.94	1.8749	1.2269	0.0190
Gabon	5351.55	12402.88	0.1093	-0.0414	0.0207
Gambia	1266.55	1236.29	-0.6780	-0.7854	-0.0006
Germany	12944.22	34519.98	1.5726	1.4323	0.0242
Ghana	2114.91	2522.37	-0.5145	-0.6998	0.0043
Greece	8588.25	23698.65	0.7331	0.7112	0.0251
Guatemala	2889.37	4235.90	-0.3652	-0.5856	0.0094
Guinea	1590.00	958.320	-0.6156	-0.8040	-0.0123
Guinea-Bissau	1230.59	906.67	-0.6849	-0.8074	-0.0074
Honduras	2108.75	2919.84	-0.5157	-0.6733	0.0080
Hong Kong	6777.86	38568.79	0.3842	1.7021	0.0433
Hungary	4940.08	18852.01	0.0300	0.3883	0.0332
Iceland	14466.64	31921.62	1.8660	1.2591	0.0195
India	1222.28	3601.68	-0.6865	-0.6278	0.0267
Indonesia	825.20	4339.49	-0.763	-0.5787	0.0413
Iran	3028.48	11818.47	-0.3384	-0.0803	0.0338
Ireland	8125.97	36704.62	0.6440	1.5778	0.0375
Israel	11729.06	25081.19	1.3384	0.8034	0.0187
Italy	11089.52	29089.05	1.2152	1.0704	0.0238
Jamaica	5474.39	5078.14	0.1330	-0.5295	-0.0018
Japan	11451.39	30427.21	1.2849	1.1596	0.0241
Jordan	2702.27	5092.50	-0.4013	-0.5285	0.0156

Table 1: First Part of the Countries in the Data Set

For the beginning and end of the sample, 1970 and 2011, both absolute and standardized values of income per capita are given. Absolute values are expressed in PPP 2005 USD; standardization is carried out by subtraction of the mean and division by the standard deviation. The growth rate is the average yearly growth rate for the country based on the 1970 and 2011 absolute values.

Country	Absolute Income per Cap. 1970	2011	Stand. Income Per Cap. 1970	2011	Growth per Year (1970-2011)
Kenya	1474.72	1297.57	-0.6379	-0.7814	-0.0031
Korea (Rep.)	1903.57	27522.30	-0.5552	0.9660	0.0673
Laos	654.52	2623.87	-0.7959	-0.6930	0.0344
Lebanon	5189.38	13158.62	0.0780	0.8900	0.0230
Lesotho	536.82	1487.82	-0.8186	-0.7687	0.0252
Liberia	1596.03	474.47	-0.6145	-0.8362	-0.0292
Luxembourg	22242.02	78130.59	3.3645	4.3381	0.0311
Macao	5327.00	69471.51	0.1046	3.7611	0.0646
Madagascar	1327.29	759.41	-0.6663	-0.8172	-0.0135
Malawi	774.91	802.26	-0.7727	-0.8144	0.0008
Malaysia	2743.98	13468.81	-0.3932	0.0296	0.0396
Maldives	1108.23	10343.66	-0.7085	-0.1786	0.0560
Mali	452.41	941.06	-0.8349	-0.8051	0.0180
Malta	6220.10	23993.08	0.2767	0.7309	0.0335
Mauritania	1665.12	2615.75	-0.6012	-0.6935	0.0111
Mauritius	3806.06	9645.06	-0.1886	-0.2252	0.0229
Mexico	6929.52	12709.82	0.4134	-0.0210	0.0149
Mongolia	958.07	5219.47	-0.7374	-0.5200	0.0422
Morocco	1914.60	3647.45	-0.5531	-0.6248	0.0158
Mozambique	408.34	817.70	-0.8434	-0.8133	0.0171
Namibia	4142.39	5146.14	-0.1237	-0.5249	0.0053
Nepal	754.17	1185.38	-0.7767	-0.7888	0.0111
Netherlands	14861.05	38054.85	1.9420	1.6678	0.0232
New Zealand	14157.92	26666.53	1.8065	0.9090	0.0156
Niger	1030.42	522.560	-0.7235	-0.8330	-0.0164
Pakistan	1453.33	2472.89	-0.6420	-0.7031	0.0130
Paraguay	1815.40	4351.30	-0.5722	-0.5779	0.0216
Panama	4630.29	12154.75	-0.0297	-0.0579	0.0238
Peru	3357.04	8923.98	-0.2751	-0.2732	0.0241
Philippines	2076.39	3521.06	-0.5219	-0.6332	0.0130
Poland	4616.73	18430.43	-0.0323	0.3602	0.0343
Portugal	6807.18	22289.90	0.3898	0.6174	0.0294
Romania	2526.24	13574.31	-0.4352	0.0366	0.0419
Rwanda	971.20	1201.50	-0.7349	-0.7878	0.0052
Senegal	1633.78	1411.72	-0.6072	-0.7738	-0.0036
Sierra Leone	1182.68	867.03	-0.6941	-0.8101	-0.0075
Singapore	5262.33	51643.66	0.0921	2.5732	0.0573
South Africa	5312.42	8457.45	0.1018	-0.3043	0.0114
Spain	9549.38	28740.77	0.9183	1.0472	0.0272
Sri Lanka	2560.14	4701.08	-0.4287	-0.5546	0.0149
Sudan (pre-2011)	1010.46	2373.99	-0.7273	-0.7096	0.0211
Suriname	4156.31	6699.65	-0.1210	-0.4214	0.0117
Swaziland	1504.57	4239.25	-0.6321	-0.5854	0.0256
Sweden	16515.69	36100.79	2.2609	1.5376	0.0193
Switzerland	23658.73	44823.64	3.6376	2.1188	0.0157
Syria	2743.49	3919.02	-0.3933	-0.6067	0.0087
Taiwan	3770.13	28413.56	-0.1955	1.0254	0.0505
Tanzania	1287.40	1269.39	-0.6740	-0.7832	-0.0003
Thailand	1982.10	8491.04	-0.5401	-0.3021	0.0361
Togo	1082.39	946.69	-0.7135	-0.8047	-0.0033
Trinidad & Tobago	9203.12	20196.31	0.8516	0.4779	0.0194
Tunisia	2200.01	6632.04	-0.4981	-0.4259	0.0273
Turkey	5732.40	14437.29	0.1827	0.0941	0.0228
Uganda	985.33	1187.03	-0.7322	-0.7887	0.0046
United Kingdom	13004.91	32259.81	1.5843	1.2817	0.0224
United States	20494.50	42646.21	3.0277	1.9737	0.0180
Uruguay	7049.31	12625.06	0.4365	-0.0266	0.0143
Vietnam	700.06	3447.77	-0.7872	-0.6381	0.0397
Zambia	3873.56	2051.71	-0.1755	-0.7311	-0.0154
Zimbabwe	2128.35	4347.79	-0.5119	-0.5781	0.0176
Sample Mean	4784.40	13024.30	0	0	0.0272
Sample Stand. Deviation	5188.72	15008.01	1	1	0.2449

Table 2: Second Part of the Countries in the Data Set

For the beginning and end of the sample, 1970 and 2011, both absolute and standardized values of income per capita are given. Absolute values are expressed in PPP 2005 USD; standardization is carried out by subtraction of the mean and division by the standard deviation. The growth rate is the average yearly growth rate for the country based on the 1970 and 2011 absolute values.